# METHOD OF MEASURING SHUNT RESISTANCE IN PHOTODIODES

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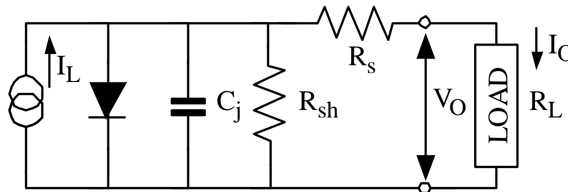
***Abstract - A method of measuring the shunt resistance of diodes, specifically photodiodes, is examined and the procedure of how the method is implemented using LabVIEW is detailed. Rudimentary comparison with other accepted methods in industry is done. The uncertainty analysis of the measurements using the specified method is given.***

## Introduction

A photodiode has two resistance components in the equivalent circuit [1] shown in Fig. 1, the series and shunt resistance.




$I_L$ : Current generated by the incident light
$C_j$ : Junction Capacitance
$R_{sh}$ : Shunt resistance
$R_s$ : Series resistance
$R_L$ : Load resistance
$I_O$ : Output current
$V_O$ : Output voltage

Fig. 1: Photodiode equivalent circuit.

The series resistor is in series with the photocurrent source, and the shunt resistor is in parallel with the photocurrent source. In instruments using photodiodes (or any other diode for that matter), it is important to know the value of the shunt resistance, so an appropriate load can be placed across the terminals of the photodiode such that a significant percentage of the generated photocurrent reaches the load. The shunt resistance is defined to be the inverse slope of the I-V curve at the 0 V point. To make the curve consistent with the theoretical model, the light input must be constant. Calculating the slope of the curve at the 0 V point is time-intensive, so the generally accepted practice in the industry is to measure the current $I$ at $V = 10$ mV, and apply Ohm's law to find the shunt resistance $R_{sh}$.

$$R_{sh} = \frac{dV}{dI} \tag{1}$$

Knowing the value of the shunt resistance is important because of the nature of electronic current to flow in the path of least resistance. In highly sensitive measurements, where large resistors are used in the feedback of an operational amplifier, it is essential to have high shunt resistance so that very small photocurrents can be measured. At the same time with high current gains, low voltage gains can be achieved if the shunt resistance is large. In addition, for applications such as trap detectors [2], which employ several photodiodes connected in parallel, the resultant noise performance of the trap detector is improved if the photodetectors have similar shunt resistances.

This paper will propose a method to easily calculate the slope of the line at the 0 V point, and compare it to the results derived from the 10 mV method and compare the uncertainties.

## Procedure

The instrument used to measure and plot the I-V curve of the photodiodes was a HP 4145A Semiconductor Parameter Analyzer [3] connected to a PC via a GPIB cable. LabVIEW 5.1 was used to communicate with the HP 4145A. A program was written to set software parameters for the I-V curve measurements and to download the data of the measurements. The data from the measurements were then analyzed mathematically; details of which will be provided later in the paper.

The photodiode in question was clipped to the leads of the probe, and placed in a HP 16058A test fixture, which blocked light and electromagnetic interference.

Previously, the shunt resistance of the photodiode was determined by fitting a line tangent to the 0 V point of the curve by eye using a line-positioning feature of the HP 4145A, and reading the inverse of the slope off the display of the 4145A. Numerically fitting a certain portion of the curve around the 0 V area with a least-squares linear fit can do this. That eliminates a significant portion of uncertainty stemming from operator placement of the tangent line. The linear fit method was chosen rather than numerically deriving the slope from the 0 V point because numerical derivations are sensitive to fluctuations in the curve. Owing to the experimental nature of the data, significant fluctuations were present, enough to introduce large errors if numerical derivation was used. Some of the photodiodes exhibited jagged curves, probably resulting from electromagnetic interference. For those curves, a fourth-order polynomial was fitted to the experimental data, and the slope was obtained from the result. A $4^{th}$ order polynomial was chosen to allow enough flexibility to match the shape of the curve, but not so much that the curve fit follows the deviations.

In fitting the straight line, two considerations need to be addressed. The data set for the linear fit needs to contain enough points so that the influence of the fluctuations of the experimental data from the theoretical line is minimized in the linear fit. The domain of the set should not be so large that the fitted line resembles more of a chord of a circle than the tangent. It was arbitrarily decided that the set for the linear fit should range from -4 mV to 4 mV and encompass eight data points.

The +10 mV and -10 mV measurements were performed by locating the current that corresponds to the ±10 mV points, respectively, and applying Ohm's law to obtain the resistance.

## Data/Observations

Initially, when the shunt resistance of a silicon photodiode was measured without pause between measurements, it was noticed that the resistance steadily increased. An example of this is shown in Fig. 2.

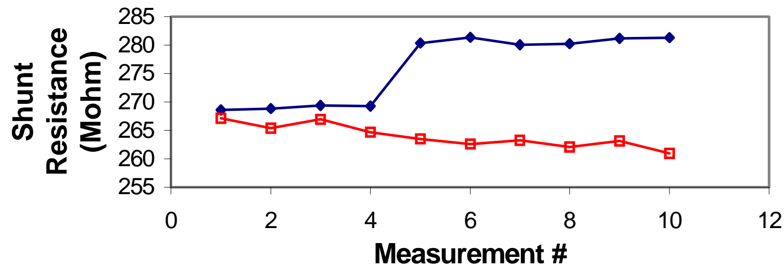


Fig. 2: Comparison between measurements done in succession and measurements spaced by five minutes. The top line indicates data that was measured in succession. The bottom line is data that was measured in intervals of five minutes.

Apparently, the effect occurs because as the current flows through the photodiode, the temperature of the photodiode increases. The shunt resistance is strongly temperature-dependent [4, 5]. Spacing the measurements by an arbitrarily chosen time of approximately five minutes eliminated this effect. A Fluke 52 thermocouple was used in an attempt to judge the effect of temperature on the shunt resistance values, but the resolution of the thermocouple was greater than the magnitude of the temperature change of the photodiode, so the contribution of temperature variation to the uncertainty is unknown.

In the case of the ±10 mV measurements, for the results to be consistent, the basis of the curve needs to be consistent. For this experiment, that means the 0 V point should also correspond to the 0 A point. This was not always the case as shown in Fig. 3, probably because of light leakage around the test fixture or a large time constant in the measuring circuit at very low currents.

Therefore, the resulting shunt resistances from the ±10 mV measurements were in error. The biased +10 mV result ranged from 25 % lower to 25 % higher than that of the linear method. On the opposite end, the -10 mV result ranged from 17 % lower to 40 % higher than that of the linear method, but usually was complementary to that of the +10 mV deviation. So when the average of both ends was calculated, a value not too far from the linear method was obtained. The curve could be corrected by subtracting the magnitude of the voltage bias. Calculations were done to eliminate the bias, as shown in Fig. 4.

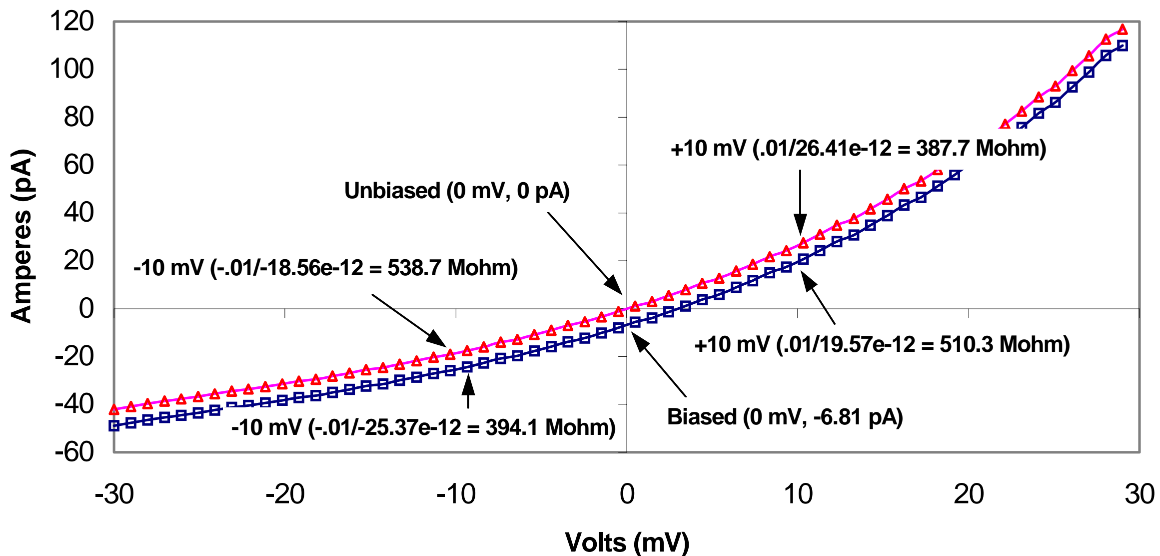


Fig. 3: This photodiode I-V curve shows the original, biased line on the bottom and the corrected, unbiased line on the top. The adjusted ±10 mV resistance values can be seen on the graph.

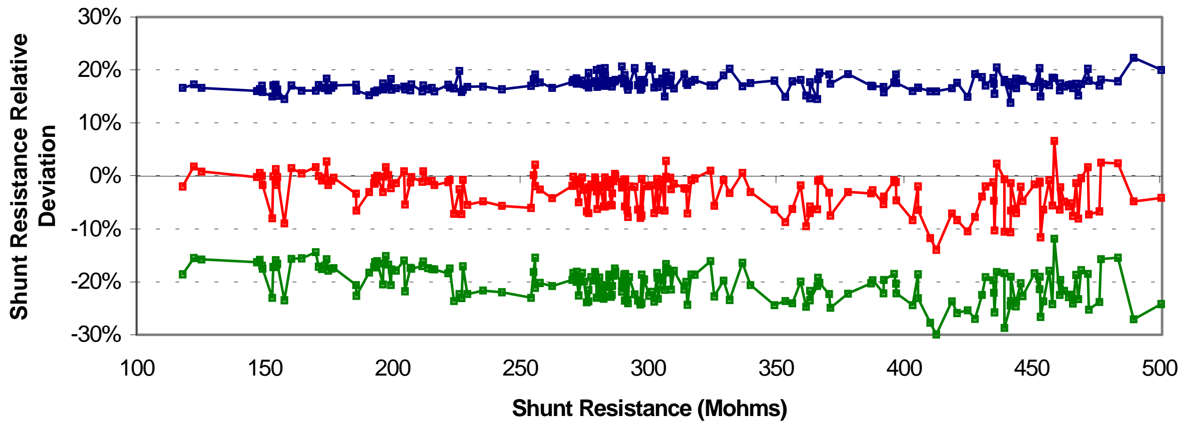



Fig. 4: The top line is the relative deviation of the +10 mV shunt resistance values from the nominal values. The bottom line is the relative deviation of the -10 mV shunt resistance values from the nominal values. The center line is the average relative deviation of the ±10 mV results from the nominal values.

The result of eliminating the voltage bias from the experimental data found that the +10 mV ranged from 13 % to 22 % lower than the actual value, with an average of (16.6 ± 1.4) %. For the -10 mV results, the range was 12 % to 30 % higher than the actual value, with an average of (20 ± 3) %. The average of the combined values of the ±10 mV results differs from the quantity recovered using the linear method by ≈ -3 % ± 3 %. This may prove to be an alternative method, easier than the linear method if the requirement for accuracy is not too high.

## Uncertainty Analysis

There was one primary thrust for uncertainty analysis in this project. Since the resistance value is garnered from the slope of the linear fit, to determine the error in the resistance, it was necessary to determine the error in the slope of the linear fit. If the expression for the linear fit is

$$\hat{y} = \beta_1 x + \beta_0 \qquad (2)$$

The uncertainty in the slope, $\beta_1$ is given to be [6]

$$\sigma_{\beta_1} = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{(n-2)\Sigma(x_i - \bar{x})^2}} \qquad (3)$$

The shunt resistance is the inverse of the slope. Therefore, the maximum and minimum resistances calculated from the maximum and minimum slope are not of equal distance from the mean. To conservatively estimate the uncertainty, the larger of the differences was chosen to represent the uncertainty in the final result.

For the polynomial curve fit, the idea is the same. The uncertainty in each of the six coefficients is calculated using [6]

$$\sigma_{\beta_i} = \sigma\sqrt{c_{ii}} \qquad (4)$$

where $c_{ii}$ are the diagonal elements of the $(X'X)^{-1}$ matrix, with the X matrix being

$$X = \begin{bmatrix} x_1^4 & x_1^3 & x_1^2 & x_1 & 1 \\ x_2^4 & x_2^3 & x_2^2 & x_2 & 1 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ x_i^4 & x_i^3 & x_i^2 & x_i & 1 \end{bmatrix} \tag{5}$$

$$\sigma = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{(n-2)}} \tag{6}$$

After all the uncertainties in the coefficients were obtained, 32 lines were formed employing the maximum and minimum of each coefficient in every combination. A linear fit was applied to each of these 32 lines, and the shunt resistance (as well as its uncertainty) was calculated. The overall maximum and minimum shunt resistance values were used as endpoints of the range of possible shunt resistances, and the greatest difference from the nominal shunt resistance value was assigned to be the overall uncertainty of the nominal shunt resistance.

The dominating factor of the uncertainty was the signal-to-noise ratio (SNR). Diodes with high SNR resulted in the lowest uncertainty. An example of a photodiode with high SNR is shown in Fig. 5. Using the techniques outlined earlier, resistances calculated with smooth I-V curves have relative uncertainties between 1 % to 3 %. As a comparison, particular photodiodes were measured 10 to 20 times, and the standard deviation of those measurements agrees with the uncertainty stemming from the mathematical analysis.

Photodiodes with low SNR exhibit visible fluctuations from the curve expected by theory. The exact cause is unknown, but electromagnetic interference is suspected. An example of a photodiode with low SNR is shown in Figs. 6 and 7. For the noisy I-V curves, the uncertainties were in the range of 20 % to 30 %. However, the shunt resistances calculated with the $4^{th}$ degree polynomial fit had uncertainties of 7 % to 12 %. The uncertainty of the 'raw' fit overlapped that of the $4^{th}$ degree fit, but the inverse is not true; the uncertainty of the majority of the $4^{th}$ degree fit result does not contain the value of the raw fit within its range.

The uncertainty of the shunt resistance values of the ±10 mV results was computed by calculating the standard deviation of the measurements. The uncertainty of those results was similar to the results recovered using the linear fit method.

## Application

In the selection of photodiodes for use in trap detectors, it is desirable to minimize the range of shunt resistance values in each trap detector because the photodiodes will be wired in parallel. The noise performance of trap detector preamplifiers is improved if the shunt resistances of each photodiode are similar to each other.

In a practical test of the shunt resistance linear fit measurement method, photodiodes were selected for 13 trap detectors. A final grouping of 66 photodiodes was selected from a pool of 71 photodiodes that minimized the range of photodiode shunt resistances in each group. The measurement results were in a range of 150 MΩ to 500 MΩ. With the measurement data, it was possible to make the selections with a maximum range per group of 179 MΩ and minimum range of 4 MΩ with an average range of 85 MΩ. This can be compared with a worst-case scenario of maximum range of 350 MΩ.

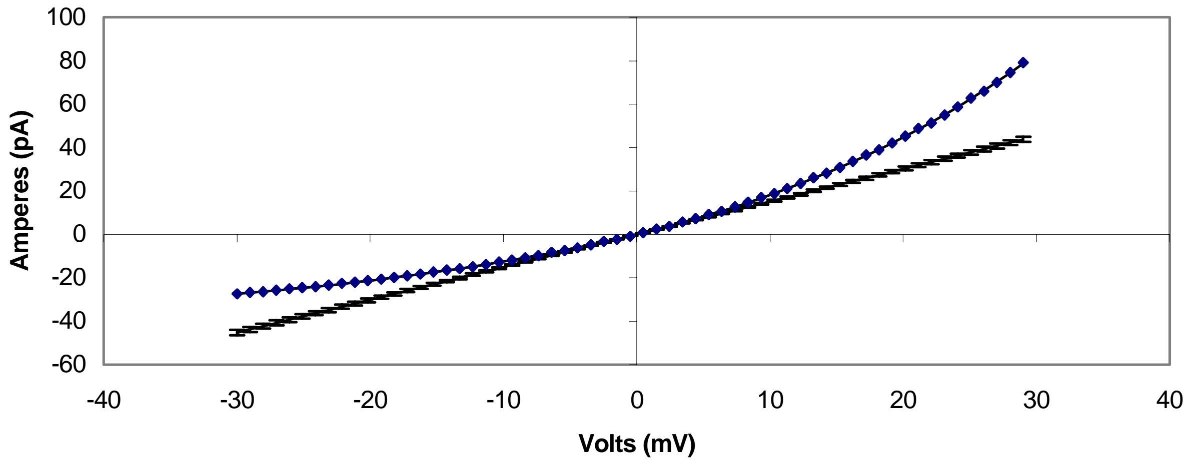



Fig. 5: I-V curve of Si photodiode with high SNR. The points are the actual data values. The line connecting the points is the curve-fitted line using a 4th order polynomial. The straight line is the linear fit used to find the slope, thus the shunt resistance. The error bars show the uncertainty of the linear fit.

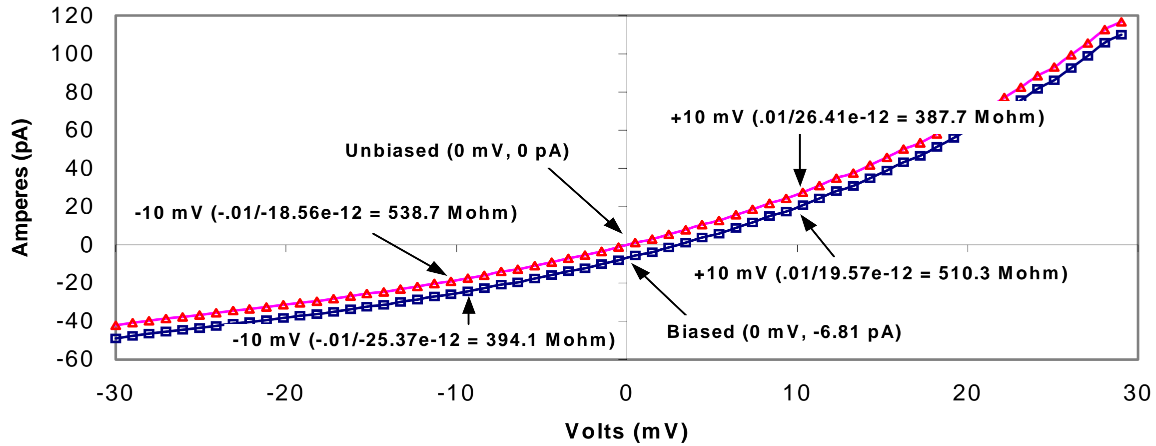



Fig. 6: Si photodiode I-V curve with low SNR. The experimental data points are connected by a line. The straight line is the linear fit of the experimental data. The error bars show the uncertainty in the fitted line.

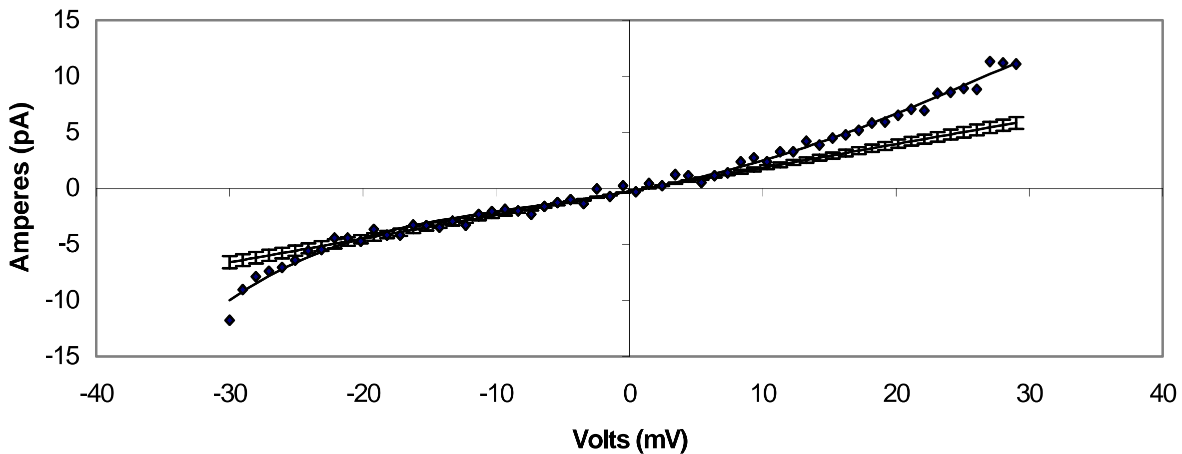



Fig. 7: The experimental data is the same as that of Fig. 6. The curve is the 4th order polynomial that is fitted to the experimental data. The straight line is the line fitted to the curve fit. The error bars represent the uncertainty of the shunt resistance values.

## Conclusions

The +10 mV measurement method, generally used in industry, does not result in particularly accurate measurements. A preliminary finding was that even if the voltage bias was accounted for, the measurement is not consistent. Further investigation can perhaps find out why this is so. The observed symmetry of the +10 mV and -10 mV measurements around the actual resistance is important. This symmetry allows the average of the +10 mV and -10 mV measurement results to be close to the least squares linear fit resistance. The disadvantage of the ±10 mV method is that several measurements are required to get the standard deviation. The advantage is that this method is easier than the mathematical calculations necessary to obtain a least squares linear fit. In addition, it has the added benefit of eliminating the influence of a voltage bias other than zero. Further investigations shall have to be made before conclusions can be drawn on the use of $4^{th}$ order polynomial fitting. The measurements were successfully employed to make groupings of photodiodes with similar shunt resistances for use in trap detectors. Further work and findings in shunt resistance measurements at NIST will be reported in the future.

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[3] Certain commercial equipment, instruments, software, or materials are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

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